Pearson

# Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Mechanics M5 (6681/01)

edexcel

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# PEARSON EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

**General Principles for Mechanics Marking**

*(But note that specific mark schemes may sometimes override these general principles)*

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side

| Question Number | Scheme | Marks |
|---|---|---|
| **1** | $b = 2a - 1$<br>$(6\mathbf{i} - 2\mathbf{j}).((5-a)\mathbf{i} + (9-b)\mathbf{j}) = \frac{1}{2} \times 0.08 \times 10^2$<br>$-3a + b = -4$<br>$a = 3,\ b = 5$ | B1<br>M1 A2<br>**DM1**A1,A1 |
| | **ALTERNATIVE**<br>$(6\mathbf{i} - 2\mathbf{j}).\frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j}) = 0.08a$<br>$a = 5\sqrt{5}$<br>$10^2 = 2 \times 5\sqrt{5} \times s$<br>$s = 2\sqrt{5}$<br>$\mathbf{s} = 2(\mathbf{i} + 2\mathbf{j})$<br>$a = 3,\ b = 5$ | M1<br>A1<br>**DM**1<br>A1<br>B1<br>A1 A1 |
| | **ALTERNATIVE**<br>$\lambda(\mathbf{i} + 2\mathbf{j})$<br>$(6\mathbf{i} - 2\mathbf{j}).\lambda(\mathbf{i} + 2\mathbf{j}) = \frac{1}{2} 0.08 \times 10^2$<br>$6\lambda - 4\lambda = 4 \Rightarrow \lambda = 2$<br>$(2\mathbf{i} + 4\mathbf{j}) = (5-a)\mathbf{i} + (9-b)\mathbf{j})$<br>$a = 3,\ b = 5$ | B1<br>M1A2<br>**DM**1<br>A1 A1<br>**[7]** |
| | **Notes for Question 1** | |
| | **Using Work-energy**<br>B1 for $b = 2a - 1$<br>First M1 for use of work-energy principle with usual rules<br>A2 for a correct equn in any form with dot product evaluated<br>Second DM1 for solving two simultaneous equation, for either a or b<br>Third A1 for $a = 3$<br>Fourth A1 for $b = 5$<br>**See Alternative**<br><br>**ALTERNATIVE using force - accln**<br>First M1 for use of **F** = $m$**a** along the wire with usual rules<br>First A1 for correct $a$<br>Second DM1 for using $v^2 = u^2 + 2as$ along wire<br>Second A1 for $s = 2\sqrt{5}$<br>B1 for use of $y = 2x - 1$<br>Third A1 for $a = 3$<br>Fourth A1 for $b = 5$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| **2 (a)** | $2\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -19.6\mathbf{k} - \mathbf{v}$ | M1 |
| | $\dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + 0.5\mathbf{v} = -9.8\mathbf{k}$ PRINTED ANSWER | A1 (2) |
| **(b)** | Aux Eqn: $m + 0.5 = 0$ | M1 |
| | $\mathbf{v} = \mathbf{A}\mathrm{e}^{-0.5t}$ CF | A1 |
| | $\mathbf{v} = -19.6\mathbf{k}$ PI | B1 |
| | $\mathbf{v} = \mathbf{A}\mathrm{e}^{-0.5t} - 19.6\mathbf{k}$ GS | M1 |
| | $t = 0,\ \mathbf{v} = (4\mathbf{i} - 6\mathbf{j} + 11.6\mathbf{k}) \Rightarrow \mathbf{A} = (4\mathbf{i} - 6\mathbf{j} + 31.2\mathbf{k})$ | M1 |
| | $\mathbf{v} = (4\mathbf{i} - 6\mathbf{j} + 31.2\mathbf{k})\mathrm{e}^{-0.5t} - 19.6\mathbf{k}$ | A1 |
| | $t = \ln 4,\ \mathbf{v} = (4\mathbf{i} - 6\mathbf{j} + 31.2\mathbf{k})\dfrac{1}{2} - 19.6\mathbf{k}$ | M1 |
| | $= (2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})$ | A1 (8) |
| | | **(10)** |

**Notes for Question 2**

**2(a)** M1 for use of **F** = *m***a** vertically with usual rules
A1 for PRINTED ANSWER

**2(b)** First M1 for forming an auxiliary equation
First A1 for a correct CF
B1 for a correct PI
Second M1 for **v** = their CF + their PI
Third M1 for use of conditions to find the constant
Second A1 for a correct particular solution
Fourth M1 for putting $t = \ln 4$ into their solution
Third A1 for correct answer

**ALTERNATIVE using Integrating Factor**:
First M1 for multiplying through by an IF and integrating LHS
First A1 for $\mathbf{v}\mathrm{e}^{0.5t} = \int -9.8\mathrm{e}^{0.5t}\mathrm{d}t$
B1 for a correct IF seen
Second M1 for $\mathbf{v} = -19.6\mathbf{k} + \mathbf{A}\mathrm{e}^{-0.5t}$ ( Must have a constant vector)
Third M1 for use of conditions to find the constant
Second A1 for a correct particular solution
Fourth M1 for putting $t = \ln 4$ into their solution
Third A1 for correct answer

| Question Number | Scheme | Marks |
|---|---|---|
| **3a** | $\mathbf{F}_1 = \frac{1}{3}(\mathbf{i}-2\mathbf{j}+2\mathbf{k}) \times 6 = (2\mathbf{i}-4\mathbf{j}+4\mathbf{k})$ N<br>$\mathbf{F}_2 = \frac{1}{7}(3\mathbf{i}-6\mathbf{j}+2\mathbf{k}) \times 14 = (6\mathbf{i}-12\mathbf{j}+4\mathbf{k})$ N<br>$\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=\mathbf{0} \Rightarrow \mathbf{F}_3 = (-8\mathbf{i}+16\mathbf{j}-8\mathbf{k})$ N | M1 A1<br>A1<br>M1 A1(5) |
| **b** | $\mathbf{G} = (\mathbf{i}-2\mathbf{j}+3\mathbf{k}) \times (2\mathbf{i}-4\mathbf{j}+4\mathbf{k}) + (\mathbf{i}-\mathbf{j}+\mathbf{k}) \times (6\mathbf{i}-12\mathbf{j}+4\mathbf{k})$<br>$= (4\mathbf{i}+2\mathbf{j}) + (8\mathbf{i}+2\mathbf{j}-6\mathbf{k})$<br>$= (12\mathbf{i}+4\mathbf{j}-6\mathbf{k})$ Nm | M1<br>A1 either<br>A1 (3) |
| **c** | $\mathbf{R} = (\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ N | B1<br>(1) |
| **d** | $(x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) \times (\mathbf{i}+3\mathbf{j}+4\mathbf{k}) = (12\mathbf{i}+4\mathbf{j}-6\mathbf{k})$<br>$((4y-3z)\mathbf{i}+(z-4x)\mathbf{j}+(3x-y)6\mathbf{k} = (12\mathbf{i}+4\mathbf{j}-6\mathbf{k})$<br>One solution is $x=-1, y=3, z=0$<br>$\mathbf{r} = (-\mathbf{i}+3\mathbf{j})+t(\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ | M1 A1<br>A1<br>B1<br>M1 A1<br>(6) |
| | | **[15]** |
| | **Notes for question 3** | |
| **3a** | First M1 for a complete method to find either $\mathbf{F}_1$ or $\mathbf{F}_2$<br>First A1 for $\mathbf{F}_1$<br>Second A1 for $\mathbf{F}_2$<br>Second M1 for equating the sum of the 3 forces to zero and solving for $\mathbf{F}_3$<br>Third A1 for a correct $\mathbf{F}_3$ | |
| **3b** | First M1 for taking moments about $O$ (or possibly another point)<br>Allow **F** x **r**<br>First A1 for a correct cross-product (either)<br>Second A1 for $(12\mathbf{i}+4\mathbf{j}-6\mathbf{k})$ | |
| **3c** | B1 for $\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ | |
| **3d** | First M1 for taking moments about $O$ (or possibly another point)<br>$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) \times$ their $\mathbf{R}$ = their $\mathbf{G}$ (not evaluated)<br>First A1 for a correct equation (cross products not evaluated)<br>Second A1 for 3 correct sim equations.<br>B1 for a correct point on their line (this may appear in their equation)<br>Second M1 for $\mathbf{r} = \mathbf{a} + t$(their $\mathbf{R}$)<br>Second A1 for $\mathbf{r} = (-\mathbf{i}+3\mathbf{j})+t(\mathbf{i}+3\mathbf{j}+4\mathbf{k})$ or any other correct answer<br>**N.B.** Need $\mathbf{r}$ = …. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4a** | $\delta A = \frac{3x}{2}\delta x$<br>$\delta m = \frac{3x}{2}\delta x \frac{m}{12a^2} = \frac{mx\delta x}{8a^2}$<br>$\delta I = \frac{mx\delta x}{8a^2}x^2 = \frac{mx^3\delta x}{8a^2}$<br>$I_y = \int_0^{4a} \frac{mx^3 \mathrm{d}x}{8a^2} = 8ma^2$ PRINTED ANSWER | M1A1<br>M1<br>M1<br>**DM**1<br>A1 (6) |
| **4b** | $\delta I = \frac{1}{3}\frac{mx\delta x}{8a^2}\left(\frac{3x}{4}\right)^2 = \frac{3mx^3\delta x}{128a^2}$<br>$I_y = \int_0^{4a} \frac{3mx^3 \mathrm{d}x}{128a^2} = 1.5ma^2$ PRINTED ANSWER | M1 A1<br>**DM**1 A1 (4) |
| **4c** | Perp axes: $I = 1.5ma^2 + 8ma^2 = 9.5ma^2$<br>$mg\frac{8a}{3}\sin\theta = -\frac{19ma^2}{2}\ddot{\theta}$<br>For small $\theta$, $\ddot{\theta} = -\frac{16g}{57a}\theta$<br>$T = 2\pi\sqrt{\frac{57a}{16g}}$ | M1 A1<br>M1 A1<br>M1<br>A1 (6) |
| | **Notes for question 4** | |
| **4a** | First M1 for attempt at area of strip in terms of a variable and an increment<br>First A1 if it's correct<br>Second M1 for multiplying their area by correct mass per unit area<br>Third M1 for $\delta mx^2$ or other appropriate $\delta$I<br>Fourth DM1, dependent on 3rd M1 for integrating their $\delta$I between appropriate limits<br>Second A1 for PRINTED ANSWER | |
| **4b** | First M1 for using appropriate expression for $\delta$I<br>First A1 for a correct expression<br>Second DM1, dependent on1st M, for integrating their $\delta$I between appropriate limits<br>Second A1 for PRINTED ANSWER | |
| **4c** | First M1 for use of perp axes<br>First A1 for a correct MI<br>Second M1 for moments equation with usual rules<br>Second A1 for a correct equation<br>Third M1 for use of small angle approximation to obtain an SHM equation<br>Third A1 for answer. | |
| | | |

| Question Number | Scheme | Marks |
|---|---|---|
| **5a** | $\frac{1}{2}MuL = \frac{1}{3}M(2L)^2\omega$<br>$\omega = \frac{3u}{8L}$ PRINTED ANSWER | M1 A2<br>A1 (4) |
| **b** | $\frac{3Mg}{2} - Mg = ML\omega^2$<br>$\frac{Mg}{2} = ML\left(\frac{3u}{8L}\right)^2$<br>$u = \sqrt{\frac{32gL}{9}}$ | M1 A1<br>**DM**1<br>A1 (4) |
| **c** | $M(A) \quad 0 = I\ddot{\theta} \Rightarrow \ddot{\theta} = 0$<br>$(\rightarrow) \quad X = ML\ddot{\theta} = 0$ | M1<br>A1 (2) |
| **d** | $\frac{1}{2}\left(\frac{4ML^2}{3}\right)\omega^2 = MgL(1-\cos\alpha)$<br>$\frac{1}{3} = (1-\cos\alpha)$<br>$\alpha = \arccos(\frac{2}{3}) = 48^o$ or better | M1 A1A1<br>**DM**1<br>A1 (5) |
| | | **15** |
| | **Notes for question 5** | |
| **5a** | M1 for conservation of angular momentum with usual rules<br>First A1 and second A1 for each side<br>Third A1 for the answer | |
| **5b** | First M1 for resolving inwards<br>First A1 for a correct equation<br>Second DM1,dependent on previous M, for substituting for $\omega$<br>Second A1 for the answer | |
| **5c** | M1 for moments equn to show that $\ddot{\theta} = 0$<br>A1 for resolving horizontally to give zero. | |
| **5d** | First M1 for energy equation<br>First A1 for KE in terms of *M, L* and $\omega$<br>Second A1 for PE<br>Second DM1, dependent on 1st M, for sub for $\omega$ and *u* to give equn in $\alpha$ only<br>Third A1 for 48° or better (0.84 rad or better) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **6a** | $m = m_0(1+kx) \qquad \dfrac{\mathrm{d}m}{\mathrm{d}t} = m_0 kv$ | M1A1 |
| | $(m+\delta m)(v+\delta v) - mv = -mg\delta t$ | M1 |
| | $\dfrac{\mathrm{d}v}{\mathrm{d}t} + \dfrac{v}{m}\dfrac{\mathrm{d}m}{\mathrm{d}t} = -g$ | A1 |
| | $\dfrac{\mathrm{d}v}{\mathrm{d}t} + \dfrac{vm_0 kv}{m_0(1+kx)} = -g$ | M1 |
| | $\dfrac{v\mathrm{d}v}{\mathrm{d}x} + \dfrac{kv^2}{(1+kx)} = -g$ | M1 |
| | $\dfrac{\mathrm{d}(v^2)}{\mathrm{d}x} + \dfrac{2kv^2}{(1+kx)} = -2g$ printed answer | A1 (7) |
| **b** | $x = 0, v^2 = 2gh \Rightarrow 2gh = A - \dfrac{2g}{3k} \Rightarrow A = 2gh + \dfrac{2g}{3k}$ | M1A1 |
| | $v = 0 \Rightarrow \dfrac{3kA}{2g} = (1+kH)^3$ | M1 |
| | $3kh + 1 = (1+kH)^3$ | |
| | $\dfrac{3h}{7} = H$ | M1 A1 (5) |
| | | **12** |
| | **Notes for Question 6** | |
| **6a** | First M1 for differentiating $m$ wrt to $t$, and using $v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$<br>First A1 for correct expression for $\dfrac{\mathrm{d}m}{\mathrm{d}t}$ in terms of $v$<br>Second M1 for impulse-momentum principle<br>Second A1 for a correct diff equn in $m$, $v$ and $t$.<br>Third M1 for sub for $m$ and $\dfrac{\mathrm{d}m}{\mathrm{d}t}$<br>Fourth M1 for changing $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ to $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ to $\frac{1}{2}\dfrac{\mathrm{d}(v^2)}{\mathrm{d}x}$<br>Third A1 for PRINTED ANSWER | |
| **6b** | First M1 for using initial conditions<br>First A1 for a correct expression for $A$<br>Second M1 for putting $v = 0$<br>Third M1 for solving for $H$ in terms of $h$ only.<br>Second A1 for $3h/7$. | |

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